Lesson 3.

Project management and the critical path method

1 Project management

- Set of **activities** $\{1, \ldots, n\}$
- a_k = time required to accomplish activity k
- Activity j is a **predecessor** of activity k if j must be completed before k can begin
- **Due date** by which all activities must be completed
- What is the **earliest start time** for each activity? What is the **latest start time** for each activity?

Example 1. Cantor Construction is developing a one-story medical office building near the Simplexville Hospital. This construction project involves 9 activities:

k	Activity	Duration a_k (days)	Predecessor activities
1	Foundation	15	None
2	Rough plumbing	5	None
3	Concrete slab	4	1,2
4	Structural members	3	3
5	Roof	7	4
6	Rough electrical	10	4
7	Heating and air conditioning	13	2, 4
8	Walls	18	4, 6, 7
9	Interior finish	20	5, 8

The company is under contract to complete construction in 80 days. What is the earliest time that each activity can begin? What is the latest time each activity can begin?

2 Project networks

- Precedence relations would look nice on a graph...
- A **project network** is a digraph with
  - special <u>start</u> and <u>finish</u> nodes
  - one node for each activity
  - edges with length 0 connect the start node to all activities without predecessors
  - edges of length a_k connect each activity k to all activities for which it is a predecessor, <u>or</u> to the finish node if there are no such activities

Example 2. Draw the CPM project network for Example 1.

3 Earliest start times

- In Example 1, activity 3's predecessors are

- Therefore, the earliest start time for activity 3 is

- Constraints for activity 3 imposed by its predecessors correspond to paths from the start node to node 3 in the corresponding project network

⇒ The earliest start time for activity 3 is equal to

- In fact, this reasoning holds in general for any activity
- These longest paths are called **critical paths**
- The length of the critical path from the start node to the finish node is

- The nodes in the critical path from the start node to the finish node correspond to

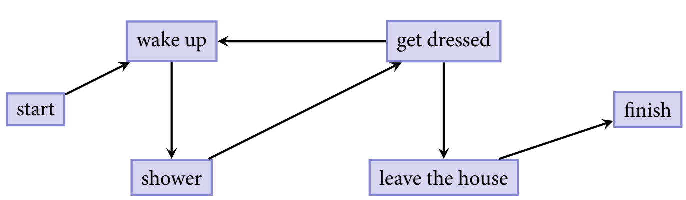
Example 3. Consider Example 1. Formulate the problem of finding the earliest time that the entire project can be completed as a shortest path problem.

4 Latest start times

- Recall that we have a due date for the project
- What is the latest time we can start each activity and still complete the project before the due date?
- Consider activity 4 in Example 1
- The path (4, 5), (5, 9), (9, finish) has a length of 30, which tells us
- The latest start time for activity 4 is equal to
- This reasoning holds in general for any activity

5 Acyclic networks and CPM

- What if a project network contains a cycle?
- For example:



- An **acyclic network** is a network that contains no directed cycles
- Well-formed project networks are acyclic and so we don't have to worry about the existence of negative cycles

Example 4. Taking this course has inspired you to optimize your breakfast routine. To make your usual breakfast, you need to complete the following activities:

k	Activity	Duration (min.)	Predecessors
1	Boil water	5	None
2	Get dishware	1	None
3	Make tea	3	1,2
4	Pour cereal	1	2
5	Fruit on cereal	2	4
6	Milk on cereal	1	4
7	Make toast	4	None
8	Butter toast	3	7

Your goal is to make your breakfast as quickly as possible. Formulate this problem as a shortest path problem.